Exercise 45

(a) If $f(x) = \sqrt{3-5x}$, use the definition of a derivative to find $f'(x)$.

(b) Find the domains of f and f'.

(c) Graph f and f' on a common screen. Compare the graphs to see whether your answer to part (a) is reasonable.

Solution

Part (a)

Use the definition of a derivative to find $f'(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3-5(x+h)}-\sqrt{3-5x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3-5x-5h}-\sqrt{3-5x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3-5x-5h}-\sqrt{3-5x}}{h} \times \frac{\sqrt{3-5x-5h}+\sqrt{3-5x}}{\sqrt{3-5x-5h}+\sqrt{3-5x}} \\
&= \lim_{h\to 0} \frac{(3-5x-5h)-(3-5x)}{h\left(\sqrt{3-5x-5h}+\sqrt{3-5x}\right)} \\
&= \lim_{h\to 0} \frac{-5h}{h\left(\sqrt{3-5x-5h}+\sqrt{3-5x}\right)} \\
&= \lim_{h\to 0} \frac{-5}{\sqrt{3-5x-5h}+\sqrt{3-5x}} \\
&= \frac{-5}{\sqrt{3-5x}+\sqrt{3-5x}} \\
&= -\frac{5}{2\sqrt{3-5x}}
\end{aligned}$$

Part (b)

The domain of $f(x) = \sqrt{3-5x}$ is

$$3 - 5x \geq 0$$

$$-5x \geq -3$$

$$x \leq \frac{3}{5},$$

whereas the domain of $f'(x)$ is

$$3 - 5x \geq 0 \quad \text{and} \quad 3 - 5x \neq 0$$

$$3 - 5x > 0$$

$$-5x > -3$$

$$x < \frac{3}{5}.$$

Part (c)

Below is a graph of $f(x)$ and $f'(x)$ versus x.